MICROTURBULENCE AND WESSELINK RADII

Department of Astronomy
University of Toronto, Canada

ABSTRACT

Using the calibration of Bell and Parsons (1974), the effects of changes in microturbulence and surface gravity throughout the cycles of δ Cephei and η Aquilae are estimated. After the changes in microturbulence have been taken into consideration, Wesselink radii for these stars are reduced 20%.

I. INTRODUCTION

Because the Wesselink technique allows the radius for a pulsating star to be determined without the use of reddening, or a temperature or luminosity scale, it can be used as a valuable check on such calibrations. Wesselink radii have recently been discussed in connection with the Cepheid mass problem (Cox, 1978; Cogan, 1978). Both these papers point out discrepancies among various methods of radius determination. In addition some techniques similar to Wesselink's method (Balona, 1977; Budding, 1977) give results which disagree with the Wesselink radii. Finally, there are indications of internal inconsistencies within the Wesselink technique itself (Evans, 1976; Gieren, 1976).

The assumption in Wesselink's method which appears the most likely to cause errors is that points of equal color are points of equal temperature. The calibrations of Bell and Parsons (1974) of B-V, which is used in most Wesselink radius determinations, allows one to check the effect of two possible causes of deviation from this equal colour--equal temperature condition. From their Table II, it is possible to estimate the effect on B-V of variations of effective gravity and microturbulence throughout a Cepheid cycle. This paper is a discussion of these effects for two stars, δ Cephei and η Aquilae. These stars were chosen because surface gravities and microturbulent velocities are available for many phases of these stars.

II. SURFACE GRAVITIES

Surface gravity curves of Cepheids reported in different studies show dissimilarities. Pel (1978) has provided data for 170 Cepheids using the Walraven photoelectric system and Kurucz (1975) model atmospheres. Parsons (1971) has similarly derived surface gravities for 48 Cepheids using 6 color photometry and his atmospheres. Only a few stars are common to both studies, but a detailed comparison of the data for η Aquilæ show disagreements in the shapes of the curves. Pel's curve is quite flat from phases $\phi = .0$ to $.6$, with a sharp peak near $\phi = .8$. This is in agreement with dynamical surface gravity curves which are determined primarily by the acceleration of the atmosphere. Parsons' curve, on the other hand has a gradually downward sloping branch from $\phi = .0$ to $.7$, and a peak at maximum light.

Figure 8 in Lub and Pel (1977) makes it clear that taking account of the variation in microturbulence as a function of phase is critical in interpreting the photometric colors in terms of surface gravity. Though this is not done in Pel's curves, rough estimates of the effects, based on the microturbulence curves in the next section show that the change in microturbulence will leave the basic form of the curve unchanged but smooth out the dip in the surface gravities at $\phi = .6$, and increase the maximum of this curve.

The purpose of this section is to estimate the effect on B-V of differences in surface gravities at points of equal color. The results from the spectral synthesis of Bell and Parsons (1974) were combined with gravity differences for various B-V colors for η Aquilae. Both estimates of surface gravity curve, Parsons' and Pel's, were used for the test. Instead of Pel's curve, the curve computed by Parsons (1971) from the acceleration and radius variation tabulated by Schwarzschild,Schwarzschild,and Adams (1948) was used. This curve and Pel's are similar in shape but Pel's curve has a smaller amplitude. For both types of surface gravity curve, the effect on B-V was only a few thousandths of a magnitude. Thus these two rather different cases both show that the distortion to B-V is inconsequential, as has been previously suggested by Woolley and Carter (1973).

For δ Cephei the surface gravity curve found by Parsons (1971) from the 6 color data also yielded negligible effect.

III. MICROTURBULENCE

Variations in microturbulence throughout Cepheid cycles have been observed, but for only a few stars have enough points been observed to give more than a suggestion of the phase dependence of this parameter. Table I summarizes the available data. The variation in microturbulent velocity has been estimated from the results of various curve of growth studies. In general the trend mentioned by Schmidt (1971a) of higher microturbulence during the steep branch of the radial velocity curve is confirmed, while nearly all stars have a small constant microturbulent velocity during the gradual branch of the radial velocity curve.

TABLE I

	Period (days)	Minimum & Maximum Microturbulent Velocity (km/sec)	Comments (v = microturbulent velocity)	Reference
TU Cas	2.14	--	v = 10 km/sec, 3 plates	(1)
SZ Tau	3.15	4.0 - 6.4	4 plates	(1)
RT Aur	3.73	3.5 - 5.0	11 plates, good phase coverage	(2)
α UMi	3.97	--	v = 5.9, 1 plate ϕ = .46	(1)
δ Cep	5.37	3.8 - 5.5	9 plates, good phase coverage	(3)
U Sgr	6.74	4.2 - 7.5*		(4)
η Aql	7.18	4.0 - 5.8 4.7 - 9.0*		(5) (4)
S Nor	9.75	5.0 - 10.0* or 9.0		(4)
β Dor	9.84	5.5 - 6.0		(6)
RX Aur	11.62	5.4 - 7.6	5 plates	(7)
X Cyg	16.39	6.2 - 7.6	2 plates	(7)
Y Oph	17.12	7.5 - 10.0*		(4)
T Mon	27.02	--	v = 9.0 km/sec, 3 plates	(7)
ℓ Car	35.54	6.0 - 7.9	no plates from ϕ = .69 to .94	(8)
SV Vul	45.10	7.6 - 9.9	4 plates	(9)

*high velocities from Schmidt's plates taken without an image tube, which give systematically higher microturbulent velocities than those with an image tube.

References

(1) Schmidt, 1974

(2) Bappu and Raghavan, 1969

(3) van Paradijs, 1971

(4) Schmidt, 1971a

(5) van Hoof and Deurinck, 1952

(6) Bell and Rodgers, 1964

(7) Schmidt, *et al*, 1974

(8) Rodgers and Bell, 1968

(9) Kraft, *et al*, 1959

Schmidt points out that his data are somewhat puzzling. His plates taken with an image tube give smaller values than those taken without, and it is not clear whether some of the large values in Table I are accurate. Most entries in Table I indicate an amplitude of microturbulence of about 2 km/sec. There is no indication that the amplitude increases with period, though the mean values do. Because no attempt has been made to adjust the data for variations on technique, such as different oscillator strengths or temperature fitting procedures, this trend should be only taken as suggestive.

It is clear that it is very important to consider variations of microturbulence in interpreting colors in terms of physical parameters (Pel, 1978; Relyea and Kurucz, 1978) and that the amplitudes of Table I will have an appreciable effect on the colors. In the next section, the results of this effect on Wesselink radii will be discussed.

IV. WESSELINK RADII

In order to assess the effect of changes in microturbulence, the results of Bell and Parsons (1974) have again been used, together with the microturbulence data from van Paradijs (1971) for δ Cephei, and van Hoof and Deurinck (1952) for η Aquilae. The corrections to the color curves resulted in a 20% decrease in the Wesselink radius in both cases.

In the case of δ Cephei, there was a further interesting result. The following internal inconsistency has been noted in Wesselink's method. If the ratios of the radii at different colors are plotted as a function of the differences of radii from the radial velocity curve, the result should be a line which passes through the point (0,1). In many, though not all cases (Evans, 1976; Gieren, 1977), the points in fact lie on a loop as the colors go from blue to red. This loop appears too large to be explained by observational errors. In the case of δ Cephei when the corrections for microturbulence were made, the loop was reduced considerably and the internal standard error of the mean radius was reduced from 4.3 solar radii to 2.5 solar radii. For η Aquilæ the reduction was less, from 4.8 solar radii to 3.8 solar radii.

V. DISCUSSION

The inclusion of the correction for microturbulence, based on the results of Bell and Parsons is important in determining Wesselink radii, though the current information on the variation of surface gravity indicates that a correction for it is not. A small surface gravity correction may be necessary for cooler stars, in the opposite sense to the microturbulence correction.

Schmidt (1971b) has made an extensive study of four Cepheids, including their line blocking. His conclusions are somewhat different from those of this paper, partly because he finds variations in gravity which are different in amplitude and phase from those used here. However, his measurements of the line blocking in B-V indicate that blocking increases when microturbulence increases. In his Figure 2b,c,d, points with high microturbulence sit above those with low-microturbulence.

Variations to Wesselink's method such as that used by Balona (1977) or Thompson's (1975) method for deriving the slope of the surface brightness-color relation should be affected by differences in microturbulence, but because the effect on B-V is a few hundredths of a magnitude, the results will be little changed. In Thompson's Figure 2, the variation of surface brightness as a function of color, there is a suggestion that the red end of the relation for several stars has more scatter than the blue end. If a check of the data reveals that the scatter is phase dependent at the red end, then it is probably due to changes in microturbulence. A range of slopes for the surface brightness--color relation from roughly 2.0 to 2.2 is possible from the graph of η Aquilae. These values are extremes, but they may explain some of the scatter in the values of the slopes from different stars. In addition if the Wesselink radii used are approximately 20% too large, the slopes will be systematically about 5% too small.

The analyses in sections II and III above indicate that a method such as Balona's (1977) will be affected by changes in microturbulence or effective gravity for less than half the period. In particular, if only the descending branch of the light curve is used, the results will be altered by these changes very little. The fact that Balona's radii are in general smaller than those of Evans (1976) confirms the results of this paper. (This is true even after the Evans radii are multiplied by 1.31/1.41, to use the current foreshortening-limb darkening correction.)

Because of the importance of uniform, detailed information about microturbulence in Cepheids, an observational program is being undertaken, but from the current information it seems that Wesselink radii of these short period stars must be reduced.

I would like to thank Alan Irwin for several stimulating discussions, and my husband for comments on the writing. This work was carried out with the financial support of Operating Grant A 5419 from the National Research Council of Canada to Dr. J. R. Percy.

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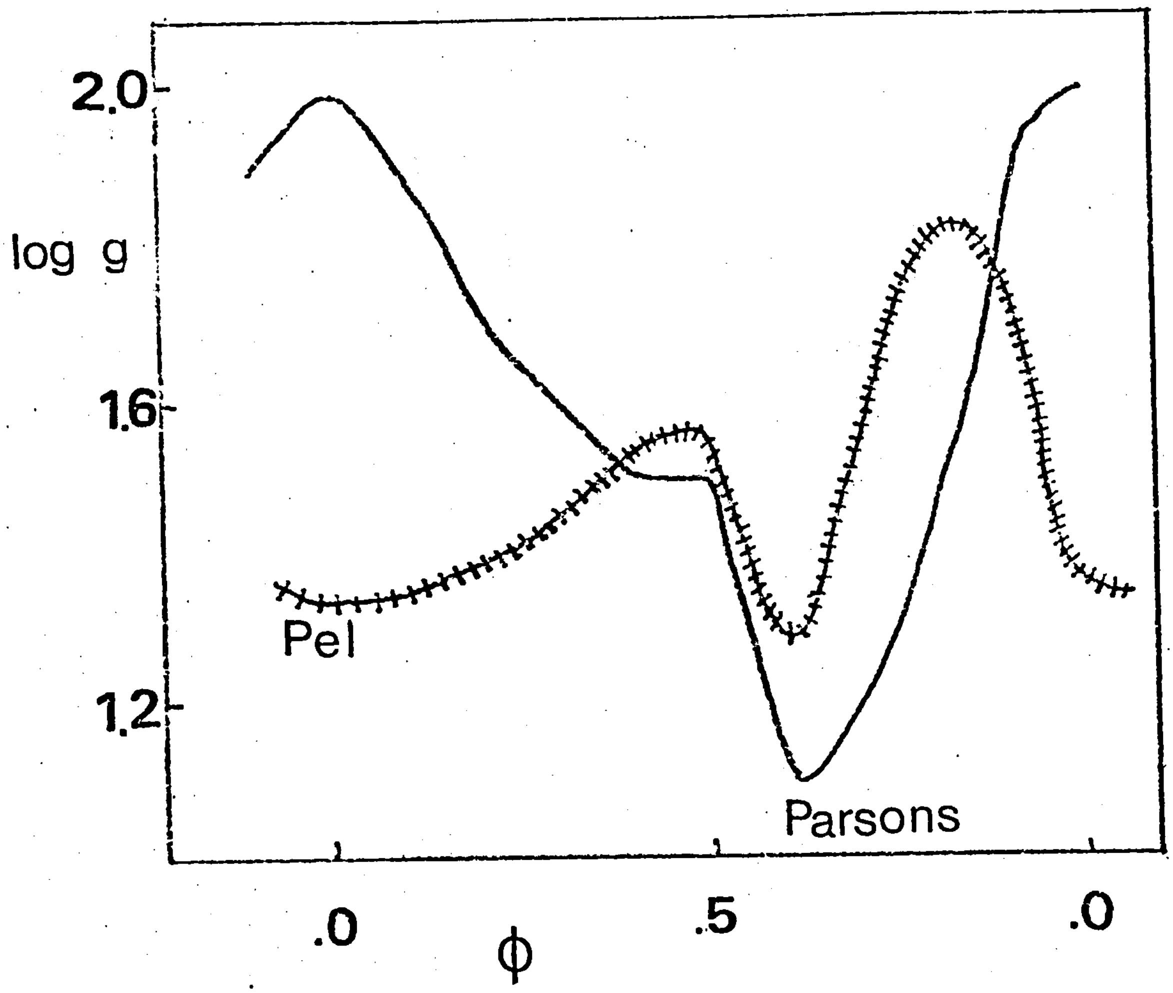

Figure 1. Gravity curves from Parsons (1971) and Pel (1978) (schematic) for η Aquilae

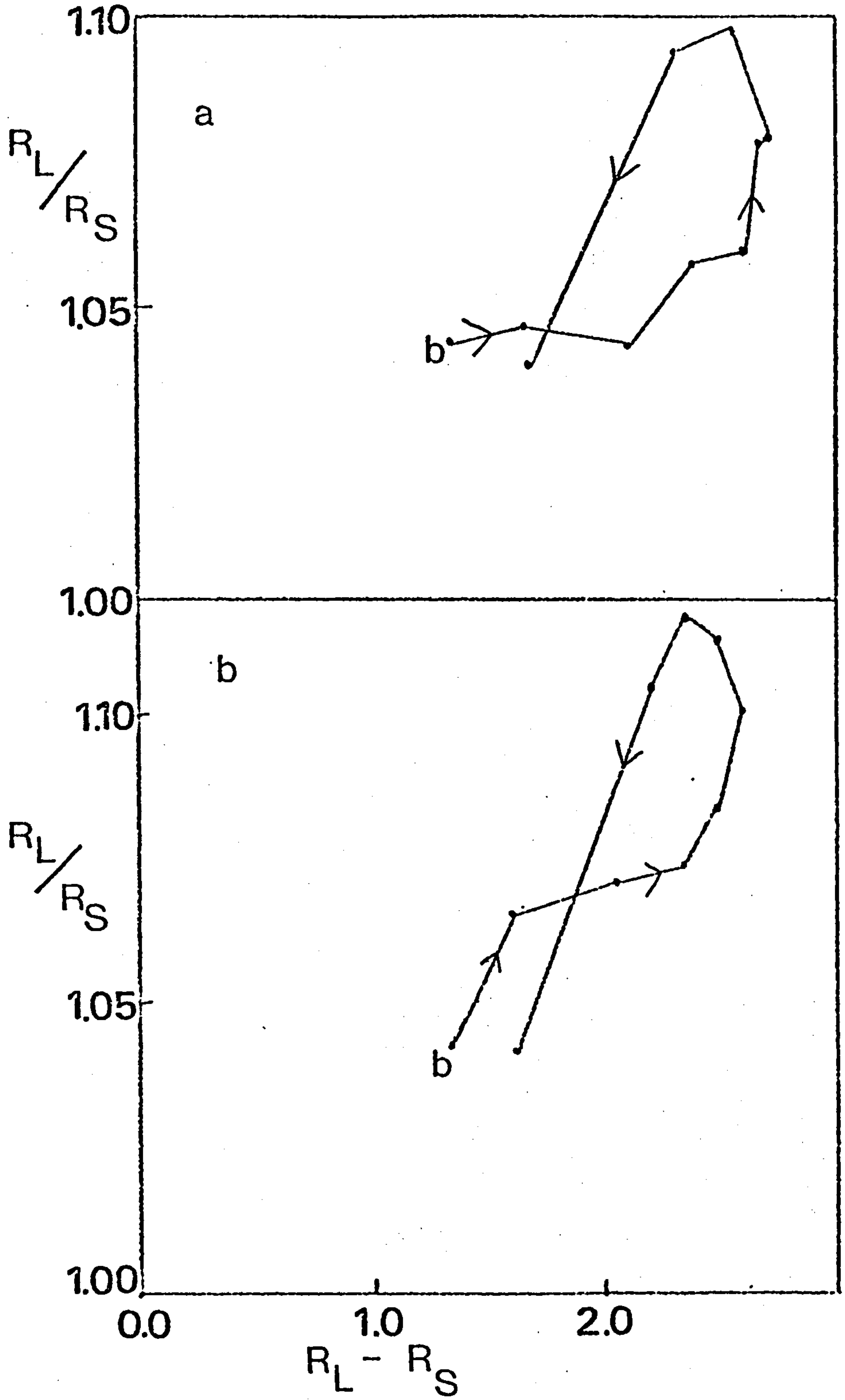

Figure 2. The ratio of the large radius to the small radius as a function of the difference in radii for δ Cephei, a) before correction for microturbulence; b) after correction. The b near the end of the curve indicates the bluest point.